# A Local Instruction Theory for Emergent Graphical Shape Thinking

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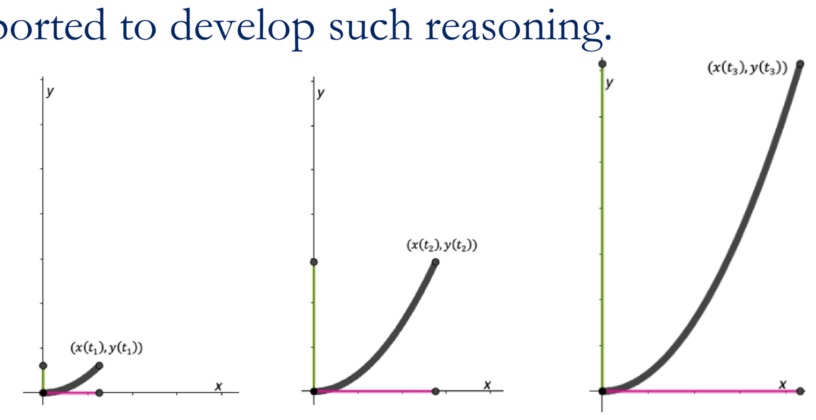
## Introduction

- Emergent graphical shape thinking (or EGST) entails conceiving of a **graph as a trace that represents a covariational relationship between two quantities' magnitudes.** Research indicates that learners must be supported to develop such reasoning.



- We describe a **local instruction theory (LIT) for how to support students' development of EGST.**

## Research Question

- How can we support eighth grade students to **develop emergent graphical shape thinking as part of their meanings** for constructing and interpreting graphs?

## Methods

- Through a series of **six small group teaching experiments, we developed a preliminary LIT** for EGST, which gave us a generalized, reasoned, and adaptable learning path that we conjectured could inform instruction around EGST.
- We conducted a final, **full-class teaching experiment with eight eighth grade students** to test our LIT. Here, we present a case study from one pair of students (Kendis and Camila) in the last experiment **to exemplify how students might engage with our LIT in practice.**

## Resources

- Scan the QR code with your phone camera or visit the URL below for access to the **digital version and additional resources.**

**tinyurl.com/Emerg-LIT-P**

## LIT for Emergent Graphical Shape Thinking

| **(M.S)** Situational quantitative and covariational reasoning | **(M.R)** Reasoning with graphical representations of covarying quantities |
|---|---|
| *(GTT) Learning prompts in Growing Triangle Task* | *(GTT) Learning prompts in Growing Triangle Task* |
| M.S.1. Construct quantities in a contextualized or decontextualized situation.<br>*(GTT) Construct base length and area as situational quantities.* | M.R.1. Consider a varying segment length as representing a quantity's magnitude.<br>*(GTT) Select a segment length representing the directional and amounts of change of triangle's varying area; conceive of segment length for base length.* |
| M.S.2. Coordinate how two quantities change in relation to each other.<br>*(GTT) Construct increasing amounts of change of area for equal base segment length changes.* | M.R.2. Consider variations in two orthogonal segment lengths on axes in a coordinate system in relation to two covarying quantities.<br>*(GTT) Construct segment lengths on axes on a coordinate plane that reflect directional and amounts of change in the triangle's area and base length.* |
| M.S.3. Develop an operative image of covariation that entails a multiplicative object.<br>*(GTT) Anticipate simultaneous covariation of area and base length by direction and amount of change.* | M.R.3. Conceive of or anticipate a point as a multiplicative object in the coordinate system simultaneously representing the two segments' magnitudes.<br>*(GTT) Construct a point in the coordinate system constrained by the segment lengths on the axes representing the segments' magnitudes.* |

**(M.E)** Emergent graphical shape thinking

*(GTT) Learning prompts in Growing Triangle Task*

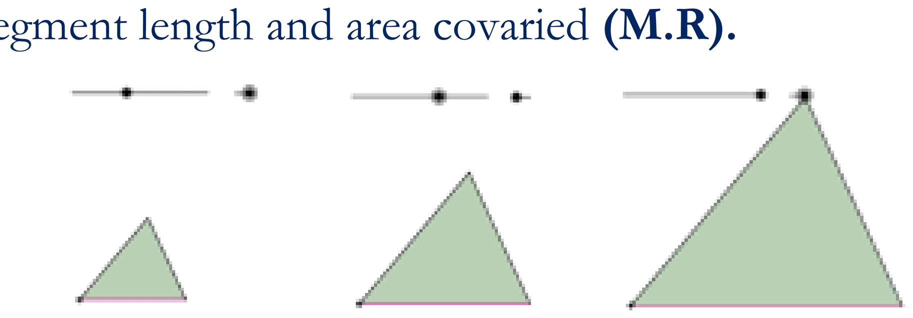
M.E.0. Conceive a point as a multiplicative object in a coordinate system (M.R.3) whose motion is constrained by the covarying quantities conceived of in the situational multiplicative object (M.S.3).
*(GTT) Describe a point's motion in the coordinate plane as representing the changes in the triangle's area and base length.*

M.E.1. Conceive of (or imagine) a graph being produced by the trace of a point (e.g., the one constructed in M.R.3) as quantities covary.
*(GTT) Construct a graph representing the relationship between the situational quantities.*

M.E.2. Consider various situations that produce the same final graph via different traces.
*(GTT) Construct graphs for Pausing and Shrinking Triangle Prompts that result in the same graph as the first via a different emergent trace.*

## Growing Triangle Task

- In the *Growing Triangle Task*, we presented students with a **dynamic applet** to examine the relationship between the quantities of area and base length **(M.S)**. Students were prompted to **construct a graph for the relationship** on whiteboards to interpret how the pink segment length and area covaried **(M.R)**.



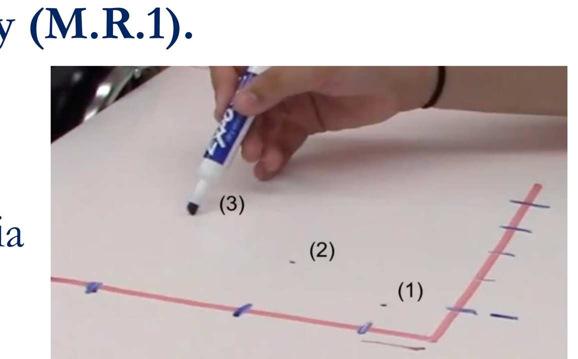
- Finally, students were presented with the *Pausing Triangle Scenario* (a video of the Growing Triangle stopping twice) and *Shrinking Triangle Scenario* (a video of the Growing Triangle changing from its maximum size to 0) **to encourage the development of their EGST (M.E)**.

## Findings (M.S & M.R)

- Kendis and Camila initially reasoned **quantitatively and covariationally** to describe the relationship between the pink segment and area **(M.S)**:

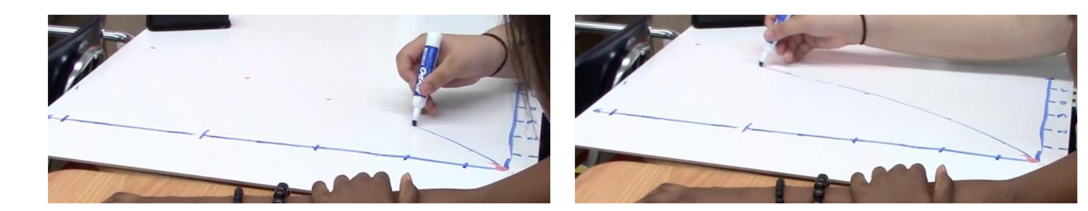
> Cam: [The] pink segment is increasing by the same amount…each time, but the area is increasing by more each time.

- Kendis identified the increasing change in the area of the triangle by **selecting a varying segment to represent the quantity (M.R.1).**
- The pair represented area on the vertical axis of a coordinate plane via **increasingly larger intervals (M.R.2)**; Camila added **points to relate the quantities (M.R.3)**.



## Findings (M.E)

- For the *Pausing Triangle Scenario*, **Camila reconstructed the Growing Triangle's graph**, explaining the original trace would pause at two locations where the triangle's growth stopped **(M.E.0, M.E.1).**



- Both students recognized that the *Pausing Triangle's* "new" graph would **reflect the same relationships as the original graph (M.E.2):**

> Cam: The graph doesn't change
> TR: Why not?
> Kend: The graph, because it's still the same points.
> Cam: It's just, they still lie on that same… [*waves over the graph*]
> Kend: Line.

- Kendis also explained the *Shrinking Triangle Scenario* would result in the **same final graph as the Growing Triangle via a different emergence (M.E.2):**

> Kend: …Now that it's decreasing, it starts at its biggest [*taps highest point*] and it goes [*traces finger along the curve from highest point to origin*] down [*smiles, looks and gestures toward graph*] on the same graph.

## Conclusions

- Our LIT posits that quantitative and covariational reasoning **(M.S)** and reasoning within a coordinate plane **(M.R)** developed a **foundation for a relationship of quantities leveraged in EGST(M.E).**
- We provide evidence that **middle school students can be supported to develop EGST** in construction and interpretation of graphs.
- We hope for this LIT to **inform future task design** across mathematics and other subject areas.

## Acknowledgements

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